Name:____________________________ Date:__________________________

Math 10/11 Enriched: Section 5.5 Solving Systems with Absolute Values

1. Solve the following equations and indicate any extraneous roots:

a) $\lvert 2x+1 \rvert = 3 - x$	b) $\lvert 3x+14 \rvert = x+2$
c) $\lvert 10x + x^2 \rvert = 24$	d) $\lvert x+3 \rvert + \lvert 5-x \rvert = 16$
e) $\lvert 2x-1 \rvert + \lvert 2x-5 \rvert - 4 = 0$	f) $\lvert x+3 \rvert + \lvert x-6 \rvert = 16$

g) $|2x+1|+|4-3x|=18$

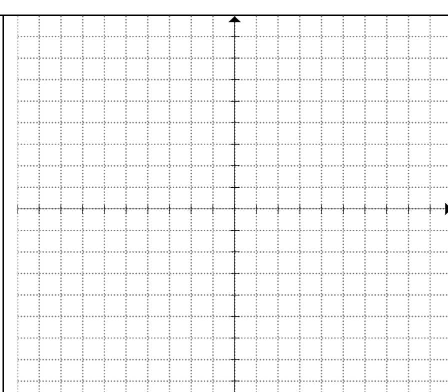
h) $|3x-1|+|3x-5|=-4$

i) $|2x+3|+|4-3x|=15$

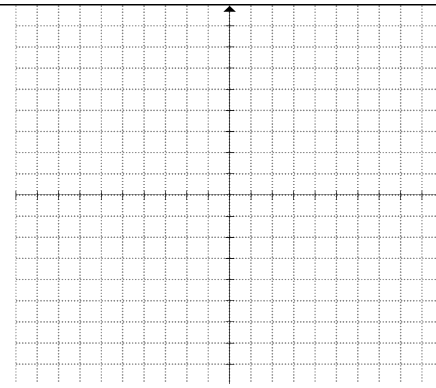
j) $|2x+3|+|3x-8|=15$

2. Solve the following equations algebraically for "x". Then use the grid of the left to graph the two sides of the equations as Y1 and Y2 with a graphing calculator. Solve for "x" graphically by finding the points of intersections. Indicate all the extranneous roots

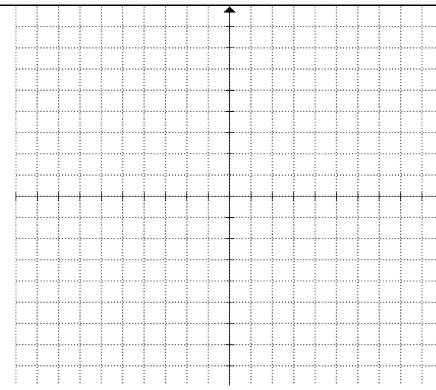
a) $|2x+5|=5$



b) $|2x+1| = 3x$

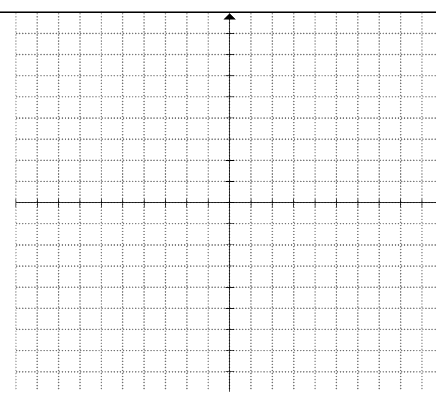


c) $|4x+10| = x+1$



b) $|2x+1| = 3x^2$

c) $|4x+10| = 2x^2+1$



3. Find the product of all real numbers “x” that satisfy the following equation: $\left|x^2 - 9x + 20\right| = \left|16 - x^2\right|$

4. What is the smallest value of “x” such that $\left|5x - 1\right| = \left|3x + 2\right|$? Express your answer as a common fraction

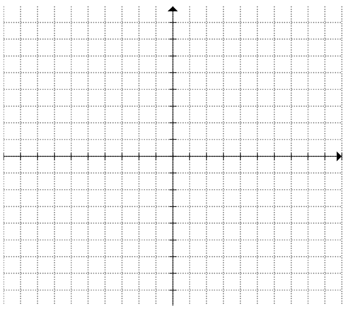
5. How many integers satisfy the following: $\left|x\right| + 1 \geq 3 \;\; and \;\; \left|x - 1\right| < 3$?

6. What are all real numbers “x” for which $\left|\left(5 - \left|x\right|\right)\right| < 14$?

7. What are both values of “x” which satisfy $x^2 + 5\left|x\right| - 6 = 0$

8. What are all the real values of 'x' which satisfy: $x+|x|=0$

9. On the grid provided, sketch the system: $y=x^2-4$ and $y=|2x|$. Find all the intersection points:



b. Determine all value(s) of "k" for which $y=x^2-4$ and $y=2|x|+k$ do not intersect.

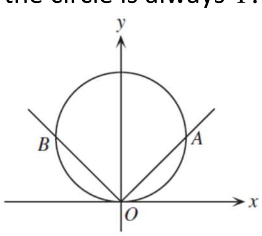
c. State the value(s) of "k" for which $y=x^2-4$ and $y=2|x|+k$ intersect in exactly two points.

How many ordered pairs of real numbers (x, y) satisfy the following system of equations?

$$x+3y=3$$
$$\big||x|-|y|\big|=1$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 8

10. A circle with its center on the y-axis intersects the graph of $y=|x|$ at the origin and exactly two other points "A" and "B" as shown in the diagram below. Prove that the ratio of the area of triangle ABO to the area of the circle is always $1:\pi$



11. What is the value of $b > 0$ for which the region bounded by both equations has an area of 72? $y = 0$ and $y = -|2x| + b$?

12. Challenge: Postive integers “a”, “b”, and “c” are chosen so that $a < b < c$, and the system of equation: $2x + y = 2003$ and $y = |x - a| + |x - b| + |x - c|$ has exactly one solution. What is the minimum value of “c”?

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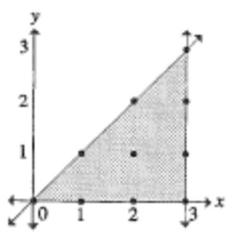
13. The graph of the three equations are given in the coordinate plane. How many ordered pairs of integers (x,y) satisfy all three equations? List all the coordinate pairs: CNML 1985 4-4

$y - |y| = 0$; $x - 3 + |x - 3| = 0$ and $y - x + |y - x| = 0$

Problem 4-4

The three equations can be rewritten as $|y| = y$, $|x-3| = -(x-3)$, and $|y-x| = -(y-x)$. From the first of these, $y \ge 0$. From the other two, $x \le 3$ and $y \le x$. From the graph of the system, we can determine that the number of lattice points (points with two integral coordinates) on the graph is $\boxed{10}$.



4-4. The graphs of the three equations

$$y - |y| = 0,$$
$$x - 3 + |x - 3| = 0, \text{ and}$$
$$y - x + |y - x| = 0$$

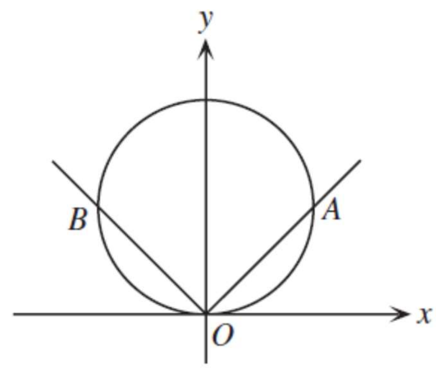
are all drawn in the coordinate plane. How many ordered pairs of integers (x,y) satisfy all three equations?

8. (a) On the grid provided in the answer booklet, sketch $y = x^2 - 4$ and $y = 2|x|$.

(b) Determine, with justification, all values of k for which $y = x^2 - 4$ and $y = 2|x| + k$ do **not** intersect.

(c) State the values of k for which $y = x^2 - 4$ and $y = 2|x| + k$ intersect in exactly two points. (Justification is not required.)

8. (a) A circle with its centre on the y-axis intersects the graph of $y = |x|$ at the origin, O, and exactly two other distinct points, A and B, as shown. Prove that the ratio of the area of triangle ABO to the area of the circle is always $1 : \pi$.



Amc 12 2008a #14

14. What is the area of the region defined by the inequality $|3x - 18| + |2y + 7| \le 3$?

(A) 3 (B) $\frac{7}{2}$ (C) 4 (D) $\frac{9}{2}$ (E) 5

Contest	Year	Number	Answer
Amc 12	2005b	7	
Euclid	2001	#8	

7. What is the area enclosed by the graph of $|3x| + |4y| = 12$?

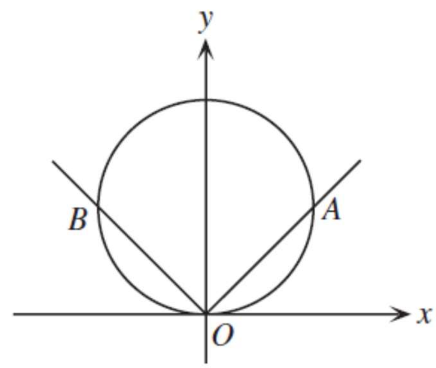
(A) 6 (B) 12 (C) 16 (D) 24 (E) 25

8. (a) On the grid provided in the answer booklet, sketch $y = x^2 - 4$ and $y = 2|x|$.

(b) Determine, with justification, all values of k for which $y = x^2 - 4$ and $y = 2|x| + k$ do **not** intersect.

(c) State the values of k for which $y = x^2 - 4$ and $y = 2|x| + k$ intersect in exactly two points. (Justification is not required.)

8. (a) A circle with its centre on the y-axis intersects the graph of $y = |x|$ at the origin, O, and exactly two other distinct points, A and B, as shown. Prove that the ratio of the area of triangle ABO to the area of the circle is always $1 : \pi$.



24. Positive integers a, b, and c are chosen so that $a < b < c$, and the system of equations

$$2x + y = 2003 \quad \text{and} \quad y = |x - a| + |x - b| + |x - c|$$

has exactly one solution. What is the minimum value of c?

(A) 668 **(B)** 669 **(C)** 1002 **(D)** 2003 **(E)** 2004

24. (C) Since the system has exactly one solution, the graphs of the two equations must intersect at exactly one point. If $x < a$, the equation $y = |x-a| + |x-b| + |x-c|$ is equivalent to $y = -3x + (a+b+c)$. By similar calculations we obtain

$$y = \begin{cases} -3x + (a+b+c), & \text{if } x < a \\ -x + (-a+b+c), & \text{if } a \le x < b \\ x + (-a-b+c), & \text{if } b \le x < c \\ 3x + (-a-b-c), & \text{if } c \le x. \end{cases}$$

Thus the graph consists of four lines with slopes -3, -1, 1, and 3, and it has corners at $(a, b+c-2a)$, $(b, c-a)$, and $(c, 2c-a-b)$.

On the other hand, the graph of $2x + y = 2003$ is a line whose slope is -2. If the graphs intersect at exactly one point, that point must be $(a, b+c-2a)$. Therefore

$$2003 = 2a + (b+c-2a) = b+c.$$

Since $b < c$, the minimum value of c is 1002.

